International Journal of Theoretical and Applied Physics, Vol. 4, No. 1, pp. 9-26 (2014)

The General Theory of Relativity, Metric Theory of Relativity and Covariant Theory of Gravitation. Axiomatization and Critical Analysis

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The axiomatization of general theory of relativity (GR) is done. The axioms of GR are compared with the axioms of the metric theory of relativity and the covariant theory of gravitation. The need to use the covariant form of the total derivative with respect to the proper time of the invariant quantities, the 4-vectors and tensors is indicated. The definition of the 4-vector of force density in Riemannian spacetime is deduced.

Keywords: general relativity; metric theory of relativity; covariant theory of gravitation; axiomatization.

PACS: 04.20.Cv, 04.50.Kd, 04.90.+e, 11.10.Cd

Establishing of axiomatic foundations is considered as an important stage in the development of any modern physical theory. This is due to the fact that from a given complete set of mutually independent axioms it is possible to deduce uniquely and unambiguously the whole theory. In addition, on the basis of the axioms it is easy to define the scope of applicability of the theory and its difference from alternative approaches.

Presented in 1915 by Albert Einstein [1] and David Hilbert [2] the equations of general theory of relativity (GR) were based on several principles and heuristic analogies, but were not axiomatized. Mathematical apparatus available in GR made it possible to solve various problems, which allowed the theory to become generally accepted model of gravitation. The problem with the axiomatization of GR became acute in mid-twentieth century, when it became clear that GR can not be quantized in the same way, as electromagnetic theory.

In GR the tensor of gravitational field is also not defined, which prevents from recognizing GR as a complete theory of gravitational field. The equations of GR predict singularities with infinite energy density, and black holes with such a magnitude of gravitation, that it must hold within itself not only the matter, but even light quanta. However, in the framework of GR it is apparently impossible to give the answer about the real existence of such exotic objects.

For theoretical foundation of GR the following principles are usually applied:

1) The principle of equivalence in different forms, including:

1.1) The equality of inertial and gravitational masses.

1.2) The equivalence of inertial and gravitational accelerations in description of phenomena in the infinitely small reference frame of the test particle.

1.3) The equivalence of the state of free falling in any gravitational field and inertial motion in the absence of a gravitational field, on condition that the instantaneous velocity of falling is equal to velocity of inertial motion.

1.4) The equivalence of the form of motion with the same initial conditions for any uncharged and non-rotating test particles in a gravitational field regardless of the structure and composition of their substance.

1.5) The equivalence of physical phenomena for the free falling in the gravitational field of the observer in his reference frame, understood as independence of the form of phenomena on the fall velocity and the location in the gravitational field.

1.6) The equivalence of effects of gravitation and deformation of spacetime; description of gravitation through the metric tensor and its derivatives with respect to coordinates and time.

2) The principle of motion along geodesics arising from 1.1), 1.3) and 1.4).

3) The principle of distortion of spacetime by matter, electromagnetic field and other non-gravitational fields.

4) The principle of linear relationship between the curvature of spacetime and the energy-momentum of matter and nongravitational fields (Hilbert-Einstein tensor equation for metric).

5) The principle of determining of force and of equations of motion through the covariant derivative of stress–energy tensor.

6) Correspondence principle: in the weak field the equations of GR turn into the classical Newton's equation of gravitation and the metric of spacetime becomes the metric of Minkowski flat spacetime.

7) The principle of covariance: physical quantities and equations of GR must be written in covariant form, independent on the choice of the reference frame.

8) The principle of least action: the equations of GR can be deduced from the variation of the four-volume integral of the Lagrangian density.

It is most convenient to measure the metric in GR by means of electromagnetic waves by determining the deflection of light rays and the effect of time dilation of time of electromagnetic clock, depending on the coordinates and time. From here we find the metric tensor that determines the gravitational field. Therefore, in GR it is assumed that the rate of change and propagation of gravitation equals the speed of light, at which the electromagnetic wave is propagating at a given point of spacetime. The speed of electromagnetic wave in the gravitational field depends on the coordinates and time and is considered as the maximum speed of transfer of interactions. Metric tensor in GR represents the gravitational field so that the covariance of the metric tensor relative to the transformations of any reference frames determines the covariance of the gravitational field.

After appearance of the metric theory of relativity (MTR) and the covariant theory of gravitation (CTG) in 2009, which were originally axiomatized [3], the need to conduct an axiomatization of GR appeared in order to compare the physical bases of these theories with a unified point of view. Axiomatization of GR can be also useful for comparison with other alternative theories of gravitation.

Analysis of GR shows that it contains two closely related components. The first of these is the general relativity of phenomena in different reference frames. This part of the theory allows to link the results of spacetime measurements of different observers and to recalculate the physical quantities of one reference frame for another. The second part of GR is the theory of gravitational field and its interaction with the matter. Both parts of GR could be completely derived from the respective systems of axioms [4]. Due of the merging of general relativity and the theory of gravitation in GR in these systems of axioms, there is one common axiom that describes the connection of the metric and the matter in the equation for calculating the metric.

Convenience of comparing GR and CTG is due to the fact that both these theories are metric tensor theories of gravitation. In contrast to GR, in CTG the Lagrangian contains additional terms, which specify directly the gravitational field [5]. The form of these terms is similar to the terms determining the electromagnetic field. At the same time the strength tensor and the stress-energy tensor of the gravitational field are introduced in consideration. In CTG the transition from the Lagrangian to the Hamiltonian is possible [6].

Axioms of general relativity in GR

1. The properties of spacetime are defined by uncharged and noninteracting test particles and waves and do not depend on the type of particles and waves.

2. The characteristic of the spacetime is the symmetric metric tensor $g_{\mu\nu}$, which depends in general on the coordinates and the time. With the help of the tensor $g_{\mu\nu}$ various invariants associated with 4-vectors and tensors are calculated.

3. The square of the interval Ds gives the square of the length of the 4-vector differential of coordinates, which does not depend on the choice of reference frame:

$$(Ds)^2 = g_{\mu\nu} Dx^{\mu} Dx^{\nu} = g'_{\mu\nu} Dx'^{\mu} Dx'^{\nu} = (Ds')^2 ,$$

where the symbol D denotes the total differential in curved spacetime.

Spatio-temporal measurement and fixing of the metric properties are carried out usually by means of electromagnetic waves, the speed of which is considered equal to the speed of light for local observers and does not depend on the speed of the radiating bodies. However, in the global coordinate reference frame in the general case the speed of electromagnetic waves depends on the coordinates and time, and decreases in the strong field. At the same time for the electromagnetic waves the interval is always zero: $Ds = 0$.

4. Physical properties of substance and fields, except the gravitational field, are specified by the corresponding stress–energy tensors. There is a mathematical function of the metric tensor $g_{\mu\nu}$ (e.g. the Hilbert-Einstein tensor on the left side of the equation for the metric) which is proportional to the total stress–energy tensor of matter and fields on the right side:

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = \frac{8\pi G}{c^4}\left(\phi_{\mu\nu} + W_{\mu\nu}\right), \tag{1}$$

where $R_{\mu\nu}$ − Ricci tensor, R − scalar curvature, G − gravitational constant, c − the speed of light, $\phi_{\mu\nu}$ − stress–energy tensor of matter, $W_{\mu\nu}$ − stress–energy tensor of electromagnetic field and other nongravitational fields. Using this equation, the connection is realized between the geometric properties of spacetime, on the one hand, and the physical properties of existing matter and non-gravitational field, on the other hand.

5. There are used additional conditions which determine the necessary for the calculation ratios for the shifts and turns of the compared reference frames, the velocities of their motion relative to each other, and taking into account the symmetry properties of reference frames.

To derive the transformations linking the differentials of the coordinates and time of any two frames of reference, we use the condition of equality of intervals $Ds = Ds'$ in axiom 3. The interval is invariant for the calculation of which in each reference frame the knowledge of the metric tensor, specified in axiom 2, is required. In addition, according to axiom 5 there should be additional relations and connections between these frames of reference. For example, the Lorentz transformations for two inertial reference frames take into account: the location and relative orientation of the reference frames; and their velocities relative to each other; the symmetry of the transformations for the axes perpendicular to the velocity of motion including the equal value of the speed of light.

The principle of equivalence can be considered as the consequence of the independence of metric on the type and properties of test particles and waves, as it is assumed in axiom 1. From axiom 4 it follows that the transition from general relativity in GR to particular relativity in special theory of relativity must be accompanied by tending to zero of the mass density and the velocity of test particles, as well as the strengths of non-gravitational fields acting on the particles. Taking in account axiom 5 it is enough to obtain all the relations of special relativity.

Above we introduced the system of axioms of general relativity in GR from our point of view. There are other attempts to justify the relativity in GR. In particular, the article [7] is devoted to the problem of axiomatic deriving of Lorentzian geometry of free fall and light propagation that underlines the space-time of general relativity from compatible conformal and projective structures on a four dimensional manifold.

Axioms of gravitational field in GR

1. The properties of the gravitational field are set by the velocity of propagation of gravitational interaction, equal to the speed of light and depending in general on coordinates and time, as well as by non-degenerate metric tensor of the second rank $g_{\mu\nu}$.

2. The gravitational field is reduced to the geometric curvature (deformation) of spacetime caused by the sources of matter and any nongravitational field. The degree of curvature of spacetime is fixed by the curvature tensor of the Riemann-Christoffel $R_{\rho\mu\sigma\nu}$ which is the function of $g_{\mu\nu}$ and its derivatives of first and second order with respect to coordinates and time. With the help of metric contraction, from the tensor $R_{\rho\mu\sigma\nu}$ the Ricci tensor $R_{\mu\nu}$ and then scalar curvature R can be obtained.

3. Gravitational acceleration is reduced to the gradients of the metric tensor $g_{\mu\nu}$, i.e. to the rate of change of the components of the metric tensor in space and time.

4. The properties of matter and non-gravitational fields are determined by the stress–energy tensor $T_{\mu\nu}=\phi_{\mu\nu}+W_{\mu\nu}$.

5. Relationship between the gravitational (metric) field, determined by the metric tensor $g_{\mu\nu}$ through the curvature of spacetime, and the matter and field is defined by the Hilbert-Einstein equations for the metric:

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} .$$

From Axiom 3 here the principle of equivalence can be deduced. The covariant derivative acting on both sides of the equation for the metric in axiom 5, converts them to zero. It defines the properties of the Hilbert-Einstein tensor, and simultaneously sets the equation of motion of matter.

Comparison of the theories of relativity

Axioms of the metric theory of relativity (MTR) are [3]:

1. The properties of the spacetime manifold in a given frame depend on the properties of test bodies and the waves, through which the spacetime measurements are fulfilled in the frame of reference. The most important property of test bodies and the waves is the speed c of their propagation, as it appears in formulas as a measure of the velocities of other bodies and the delay of information in distance measurements.

2. Geometric properties of spacetime are fixed by a relevant mathematical object, which is a function of spacetime coordinates of the reference frame. For a large class of reference frames a suitable mathematical object is the non-degenerate four-dimensional symmetric metric tensor of second rank $g_{\mu\nu}$, whose components are scalar products of unit vectors of axes in the chosen reference frame. Tensor $g_{\mu\nu}$ allows finding any invariants that are associated with 4-vectors and tensors.

3. The square of the interval $(Ds)^2$ between two close events, understood as the tensor contraction of the metric tensor $g_{\mu\nu}$ with the product of differentials of the coordinates

$Dx^{\mu} Dx^{\nu}$, is an invariant, the measure of proper time τ of the moving particle, and does not depend on the choice of reference frame:

$$(Ds)^2 = c^2 (D\tau)^2 = g_{\mu\nu} Dx^{\mu} Dx^{\nu} = g'_{\mu\nu} Dx'^{\mu} Dx'^{\nu} = (Ds')^2 .$$

The interval Ds for two close events is zero, if these events are related to the propagation of test bodies and the waves, through which the spacetime measurements and fixing of metrics are fulfilled.

4. The physical properties of any fields including the gravitational field in some frame of reference are set by the corresponding stress–energy tensors. There is a mathematical function of the metric tensor $g_{\mu\nu}$, found by certain rules and proportional to the total stress–energy tensor of fields, acting in this frame of reference. In the simplest case, such function is the tensor, in the left part of the equation for metric [8]:

$$R^{\alpha\beta} - \frac{1}{4} R g^{\alpha\beta} = \frac{8\pi G\beta}{c^4} (B^{\alpha\beta} + U^{\alpha\beta} + W^{\alpha\beta} + P^{\alpha\beta}) , \qquad (2)$$

where β − the constant coefficient depending on the type of test particles or waves, which is determined by comparison with experiment or with the formulas of classical physics in the weak-field or low-velocity approximation, and we believe that the speed of gravitation propagation is equal to the speed of light,

$B^{\alpha\beta}$ − the stress–energy tensor of acceleration field, which depends on the acceleration tensor $u_{\mu\nu}$:

$$B^{\alpha\beta} = \frac{c^2}{4\pi\eta} \left(-g^{\alpha\nu} u_{\kappa\nu} u^{\kappa\beta} + \frac{1}{4} g^{\alpha\beta} u_{\mu\nu} u^{\mu\nu} \right).$$

$U_{\mu\nu}$ – the stress–energy tensor of gravitational field. This tensor is expressed through the tensor of gravitational field $\Phi_{\mu\nu}$:

$$U^{\alpha\beta} = \frac{c^2}{4\pi G}\left(g^{\alpha\nu}\Phi_{\kappa\nu}\Phi^{\kappa\beta} - \frac{1}{4} g^{\alpha\beta}\Phi_{\mu\nu}\Phi^{\mu\nu} \right).$$

$P^{\alpha\beta}$ – the stress–energy tensor of pressure field, being found through the pressure tensor $f_{\mu\nu}$:

$$P^{\alpha\beta} = \frac{c^2}{4\pi\sigma}\left(-g^{\alpha\nu} f_{\kappa\nu} f^{\kappa\beta} + \frac{1}{4} g^{\alpha\beta} f_{\mu\nu} f^{\mu\nu} \right).$$

Here η and σ are some constants.

Equation (2) provides the link between the geometric properties of the used spacetime manifold, on the one hand, and the physical properties of the fields, on the other side. The covariant derivative acting on both sides of the equation for the metric (2), converts them to zero. It fixes the properties of the tensor in the left side of (2), and simultaneously sets the equation of matter motion under the influence of fields.

5. There are additional conditions which determine the ratios, necessary for the calculation, for the shifts and turns of the compared reference frames, the velocities of their motion relative to each other, and taking into account the symmetry properties of reference frames.

From the comparison of the axioms of general relativity in GR with the axioms of the metric theory of relativity the features of these theories follow which are listed in Table 1.

Table 1

Features of theories	**General relativity in GR**	**Metric theory of relativity**
Metric properties of spacetime:	Do not depend on the type of test particles and waves	Depend on the type of test particles and waves
Interval is equal to zero: $Ds = 0$	Only for electromagnetic waves	For all test particles and waves, which are used for the spacetime measurements and fixing of metric
Sources of energy and momentum that define metric:	Matter and any non-gravitational fields	Any field including the gravitational field
The principle of equivalence is understood as:	Equivalence of phenomena in two reference frames of small size, one of which is accelerated by the gravitational force, while the other receives the same acceleration under the action of uniformly distributed non-gravitational force of the same magnitude	Equivalence of energy-momentum: "In accelerated reference frame the metric does not locally depend on the type of the acting force causing this acceleration, but depends on the configuration of this force in spacetime of the reference frame defined by the stress-energy tensor"

Equivalence of the acceleration, due to gravitation and inertial acceleration under the action of uniformly distributed over the volume of the test body non-gravitational force of the same value, leads to the equality of gravitational and inertial masses. The homogeneity of the applied force means that in the system of small size all parts of the system are accelerated equally and the relative internal acceleration is absent. In this case, the separate elements of the test body do not put pressure on each other and behave as the test body moving by inertia in the absence of forces. The masses of bodies can be found by weighing in relation to the standard mass in a gravitational field, and the masses are proportional to the gravitational forces. This implies the independence of the forms of motion of the falling bodies on the mass and the composition of these bodies. Since at any point in the gravitational field a falling body behaves in the same way as moving by inertia (but with a change in velocity), it is assumed that in the falling body Lorentz covariance takes place. Then the Lorentz covariance should be at any point of the trajectory of the falling body and it does not depend on the velocity, and the falling observer should not reveal by inner experiments the acceleration of the movement. As a result, the equivalence principle leads to the identification of the effect of the gravitational field of a massive body with the effect of deformation of spacetime around the massive body. These are the consequences of the equivalence principle in general relativity.

In the metric theory of relativity (MTR), instead of the principle of equivalence of forces the principle of equivalence of energy-momentum is considered. Indeed, from equation (2) for the metric in the MTR we can see that the metric is completely determined by the sources in the form of stress–energy tensors of fields including the gravitational field itself [3]. Only the energy-momentum of the system is necessary to determine the metric and the equations of motion of a test body. If two different interactions have the same dependence of the energy-momentum, then the metric and the law of motion in both cases coincide. The equation of general relativity for the metric (1) differs from equation (2) for the metric in MTR by the fact that the right-hand side of (2) contains the stress–

energy tensor of gravitational field $U_{\mu\nu}$. The contribution of this tensor in weak fields is small, and the MTR metric is slightly different from the metric of general relativity. However, in strong gravitational fields the tensor can not be ignored, since there is significant self action of field on the source of field. Another difference is that in (2) there is no stress–energy tensor of matter and in the MTO metric depends only on stress–energy tensors of fields. In contrast, in GR, according to (1) we need stress–energy tensor of matter $\phi_{\mu\nu}$ to determine the metric. Besides, the scalar curvature R is in (1) with a multiplier $\frac{1}{2}$ and in MTR in relation (2) the scalar curvature has a multiplier $\frac{1}{4}$. This is due to the calibration of the cosmological constant and relativistic energy in the covariant theory of gravitation [8].

Comparison of the theories of gravitational field

Axioms of the covariant theory of gravitation (CTG) in 4-dimensional vector-tensor formalism have the form [3]:

1) The properties of gravitational field are set by the velocity of propagation of gravitational influence $c_g = c$, as well as the scalar potential ψ and the vector potential $\boldsymbol{D}$.

2) The potentials of the gravitational field can be combined into 4-vector of gravitational potential with lower covariant index:

$$D_\mu = \left(\frac{\psi}{c}, -\boldsymbol{D} \right).$$

The speed of change of the potentials in spacetime of chosen reference frame is determined by the tensor of gravitational field, which is 4-rotor of D_μ:

$$\Phi_{\mu\nu} = \nabla_\mu D_\nu - \nabla_\nu D_\mu = \partial_\mu D_\nu - \partial_\nu D_\mu ,$$

where ∇_μ denotes the covariant derivative , μ,ν – the usual 4-indices, so that in the case of Cartesian coordinates $\partial_0 = \frac{\partial}{\partial x^0} = \frac{\partial}{c\partial t}$, $\partial_1 = \frac{\partial}{\partial x^1} = \frac{\partial}{\partial x}$, $\partial_2 = \frac{\partial}{\partial x^2} = \frac{\partial}{\partial y}$, $\partial_3 = \frac{\partial}{\partial x^3} = \frac{\partial}{\partial z}$.

With an appropriate choice of field potentials, the symmetry relation of potentials holds:

$$\nabla_\rho \Phi_{\mu\nu} + \nabla_\mu \Phi_{\nu\rho} + \nabla_\nu \Phi_{\rho\mu} = \partial_\rho \Phi_{\mu\nu} + \partial_\mu \Phi_{\nu\rho} + \partial_\nu \Phi_{\rho\mu} = 0 . \qquad (3)$$

3) The properties of substance are set by density ρ_0 in the comoving frame of reference and by velocity $\boldsymbol{V}$.

4) In special relativity the quantities ρ_0 and $\boldsymbol{V}$ are combined into 4-vector of density of mass current or of momentum density:

$$J^\mu = \rho_0 u^\mu = \left(\frac{c\rho_0}{\sqrt{1-V^2/c^2}} , \frac{\boldsymbol{V}\rho_0}{\sqrt{1-V^2/c^2}} \right) = (c\rho, \boldsymbol{J}) ,$$

where $u^\mu = \left(\frac{c}{\sqrt{1-V^2/c^2}} , \frac{\boldsymbol{V}}{\sqrt{1-V^2/c^2}} \right)$ – 4-velocity of matter unit,

$\rho = \dfrac{\rho_0}{\sqrt{1 - V^2/c^2}}$ – the density of moving matter,

$\boldsymbol{J}$ – 3-vector of mass current density.

In Riemannian space 4-velocity is given by: $u^\mu = \dfrac{cDx^\mu}{Ds}$, and $J^\mu = \rho_0 u^\mu$.

5) The relation between the gravitational field and the matter can be expressed through the relationship of 4-vector of gravitational potential D^α and mass 4-current J^α, or through the connection between the tensor $\Phi^{\alpha\beta}$ and J^α:

$$g^{\rho\nu}\partial_\rho \partial_\nu D^\alpha + g^{\rho\nu}(\Gamma^\alpha_{\nu s}\partial_\rho D^s - \Gamma^s_{\rho\nu}\partial_s D^\alpha + \Gamma^\alpha_{s\rho}\partial_\nu D^s + D^s \partial_s \Gamma^\alpha_{\nu\rho}) = -\frac{4\pi G}{c^2} J^\alpha = -\nabla_\beta \Phi^{\alpha\beta}. \tag{4}$$

Features of gravitational field in GR and in the covariant theory of gravitation arising from their axioms are given in Table 2.

Table 2

Features of theories	**The theory of gravitational field in GR**	**Covariant theory of gravitation**
Gravitational field is:	Metric tensor field, which is characterized by the tensor $g_{\mu\nu}$ and its gradients in the form of Christoffel symbols	Physical vector field, which is characterized by the 4-potential D_μ and its gradients in the form of the antisymmetric

		tensor of gravitational field strengths
Gauge:	Contraction of the metric tensor in the form $g_{\mu\nu} g^{\rho\nu} = \delta^{\rho}_{\mu}$ gives the Kronecker delta δ^{ρ}_{μ}	The field 4-potentials are calibrated so that their divergence vanish
The speed of the gravitational field equals:	The speed of light	The speed of propagation of gravitation (about the speed of light) [9]
The connection between the gravitational field and the matter in the absence of other fields:	Through the Hilbert-Einstein tensor equations for the metric (1), linking the function of the metric tensor and the stress–energy tensor of matter	Through equation (4) for the potentials or strengths of the gravitational field, and for mass 4-current J^{μ}
Sources of energy and momentum that define metric:	Matter and any non-gravitational fields	Any fields including the gravitational field

Despite the difference in systems of axioms of gravitational field in GR and in CTG, we can show that the equation of motion of general relativity is a special case of equation of motion of the CTG. As it was found in [3], the material derivative with respect to proper time in general case can be written in the form of an operator using 4-velocity u^{μ} of the matter unit:

$$\frac{D}{D\tau}=u^{\mu}\nabla_{\mu}, \tag{5}$$

where the symbol D denotes the total differential in curved spacetime, and ∇_{μ} is the covariant derivative.

Operator (5) shall be applied only to 4-objects in spacetime, which are scalars, 4-vectors and 4-tensors. 4-velocity $u^{\nu}=\frac{dx^{\nu}}{d\tau}$ is determined by the 4-vector dx^{ν} and the invariant of proper time $d\tau$. If, however, we try to find 4-velocity through the coordinate value $x^{\nu}=(x^0,\, x^1,\, x^2,\, x^3)$ using (5) in the form $\frac{Dx^{\nu}}{D\tau}=u^{\mu}\nabla_{\mu}x^{\nu}$, then there is a discrepancy, because the value $x^{\nu}=(x^0,\, x^1,\, x^2,\, x^3)$ in Riemannian space is not a 4-vector.

By definition in CTG, the force density is the total rate of change of the 4-vector of density of mass 4-current J^{μ} by the proper time in Riemannian spacetime:

$$f^{\nu}=\frac{DJ^{\nu}}{D\tau}=u^{\mu}\nabla_{\mu}J^{\nu}=u^{\mu}(\partial_{\mu}J^{\nu}+\Gamma^{\nu}_{\mu\rho}J^{\rho})=\frac{dJ^{\nu}}{d\tau}+\Gamma^{\nu}_{\mu\rho}u^{\mu}J^{\rho}, \tag{6}$$

where $\Gamma^{\nu}_{\mu\rho}$ – Christoffel symbol.

On the other hand, the expression for the force density acting on the matter unit from gravitational and electromagnetic fields and pressure field is obtained by taking the covariant derivative in equation (2). Then the left side of the equation for the metric (2) gives zero, and from the right side of this equation it follows:

$$f^{\alpha} = \nabla_{\beta} B^{\alpha\beta} = -\nabla_{\beta} U^{\alpha\beta} - \nabla_{\beta} W^{\alpha\beta} - \nabla_{\beta} P^{\alpha\beta} = g^{\alpha\rho} (\Phi_{\rho\nu} J^{\nu} + F_{\rho\nu} j^{\nu} + f_{\rho\nu} J^{\nu}) . \quad (7)$$

where $F_{\rho\nu}$ – tensor of electromagnetic field strengths,

$j^{\nu} = \rho_{0q} u^{\nu}$ – electromagnetic 4-current,

ρ_{0q} – electric charge density of the matter unit in its rest referent frame.

Comparing (6) and (7) gives the equation of motion of the matter unit in the CTG under the influence of pressure and gravitational and electromagnetic forces:

$$\frac{dJ^{\alpha}}{d\tau} + \Gamma^{\alpha}_{\mu\rho} u^{\mu} J^{\rho} = g^{\alpha\rho} (\Phi_{\rho\nu} J^{\nu} + F_{\rho\nu} j^{\nu} + f_{\rho\nu} J^{\nu}) . \quad (8)$$

Equation (8) allows taking fully into account the reactive force of Meshcherskiy [10], which appears due to changes of the density of the matter unit. The mass density is part of the mass 4-current J^{α}, from which in (8) the derivative with respect to proper time is taken, which characterizes the reaction force in the mechanics of bodies with variable mass.

To move to the formula for the force in general relativity we should make the following simplifications in (8): assume $\Phi_{\rho\mu}$ is equal to zero (in GR the gravitational field is the metric field which does not have the property of self action, and therefore the gravitational field in the right-hand side of equation (1) as the source of curvature of spacetime is absent), and assume the mass density ρ_0 is constant with respect to time and volume of test particle. In GR, is also not used pressure tensor in the form, as we have defined it. Then the quantity ρ_0 in the left side of (8) can be canceled, and from the mass 4-current J^{α} it is possible to pass to the 4-velocity u^{α}:

$$\frac{du^{\alpha}}{d\tau}+\Gamma^{\alpha}_{\mu\rho}u^{\mu}u^{\rho}=\frac{Du^{\alpha}}{D\tau}=\frac{1}{\rho_0}\,g^{\alpha\rho}F_{\rho\mu}\,j^{\mu}\,. \tag{9}$$

In the simplest case the motion of matter in the absence of electromagnetic fields is considered: $F_{\rho\mu}=0$, or in the absence of charges of matter particles: $j^{\mu}=0$. Then the right side of the equation of motion (9) will be zero and there is equation $\frac{Du^{\alpha}}{D\tau}=0$, i. e. the 4-acceleration of a freely falling body in the gravitational field is absent. Taking into account the relations for the 4-velocity $u^{\alpha}=\frac{dx^{\alpha}}{d\tau}$ and for the interval $Ds=c\,D\tau$ or $ds=c\,d\tau$, we obtain the standard equation of motion of GR for the matter in the gravitational field in the form:

$$\frac{d}{ds}\left(\frac{dx^{\alpha}}{ds}\right)+\Gamma^{\alpha}_{\mu\rho}\frac{dx^{\mu}}{ds}\frac{dx^{\rho}}{ds}=0\,. \tag{10}$$

For the propagation of light it must be: $ds=c\,d\tau=0$. Consequently, in (9) the differential $d\tau$ must tend to zero. Further after multiplication in (9) by $(d\tau)^2$ we have:

$$d\tau\,d\left(\frac{dx^{\alpha}}{d\tau}\right)+\Gamma^{\alpha}_{\mu\rho}\,dx^{\mu}dx^{\rho}=\frac{(d\tau)^2}{\rho_0}\,g^{\alpha\rho}F_{\rho\mu}\,j^{\mu}\,. \tag{11}$$

For the first term in the left side of (11) we can write down:

$$d\tau\,d\left(\frac{dx^{\alpha}}{d\tau}\right)=d\tau\cdot\lim_{\tau_2\to\tau_1}\left(\frac{dx^{\alpha}(2)-dx^{\alpha}(1)}{d\tau}\right)=\lim_{\tau_2\to\tau_1}\left(dx^{\alpha}(2)-dx^{\alpha}(1)\right).$$

Setting now in (11) $d\tau = 0$, we obtain zero in right side and arrive at the following:

$$\lim_{\tau_2 \to \tau_1} \left(dx^{\alpha}(2) - dx^{\alpha}(1) \right) + \Gamma^{\alpha}_{\mu\rho}\, dx^{\mu}\, dx^{\rho} = 0 .$$

We shall choose as the proper time for the light quantum the parameter of time λ along the trajectory, marking the location of the quantum in space, and divide the above equation by the square of the differential $d\lambda$:

$$\frac{d}{d\lambda}\left(\frac{dx^{\alpha}}{d\lambda}\right) + \Gamma^{\alpha}_{\mu\rho}\frac{dx^{\mu}}{d\lambda}\frac{dx^{\rho}}{d\lambda} = 0 . \qquad (12)$$

Equation (12) represents the standard equation of motion for the light quantum in GR. As it was seen in the derivation of (10) and (12) from equation (8), the equations of motion of GR for particles and light are the consequence of the equation of motion of CTG. In this regard, again the question arises, why in the solar system are such phenomena discovered unexplained by GR, as the Pioneer anomaly [11] and flyby anomaly [12]? One of the possible explanations is given in [3], which emphasizes the difference between the equation of motion (10) of GR and the equation of motion (8) of CTG.

Thus, from the system of axioms for general relativity in GR, and the system of axioms for gravitational field in GR all the basic features of general theory of relativity can be deduced. The axioms of GR are given in the form that allows comparing them with the axioms of covariant theory of gravitation (CTG) and metric theory of relativity (MTR). As a consequence, it turns out in [4] that general relativity in GR is a special case of the MTR. As for the axioms of gravitational field, in GR the principle of geometrization of gravitation and the equivalence principle lead to the concept of metric tensor field as the field of gravitation. In CTG the gravitational field is characterized by the vector field of 4-potential and by built with its help antisymmetric tensor field of strengths of gravitational field, which consists of two components – the gravitational field strength and the torsion field. The principle of definition of the gravitational field in CTG is similar to the definition of electromagnetic field, so that the gravitational field of CTG is not less real than the electromagnetic field, with which it refers to the fundamental fields. The latter means that the electromagnetic and gravitational fields exist not only in researches available in modern science, but according to the theory of infinite nesting of matter they act at different levels of matter. In this case, the gravitational field at the level of elementary particles leads to strong gravitation, and at the macro level – to the normal gravitation [13].

The analysis of the equivalence principle in general relativity shows that it is valid only in the infinitely small regions, in which approximation of Lorentz covariance is possible. However, this approximation becomes inaccurate in large enough areas where we can not neglect the curvature of spacetime. For example, if the test particle is massive, its proper gravitational field should be considered in the equation of motion of the particle in an external gravitational field. This is because the metric of two interacting bodies in a nonlinear manner depends on the values of the metric of these bodies, taken separately from each other. Therefore general relativity, which uses in calculation the principle of equivalence and the principle of geometrization of gravitational field is only an intermediate theory on the way of building more complete theory of relativity and deeper theory of gravitational field, fully taking into account the interaction of gravitational field with matter and other fields.

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